

1 a When $h = 10$, $d = \frac{10}{5} + 6$
= 8

b When $h = 8.5$, $d = \frac{8.5}{5} + 6$
= 7.7

c The diameter of the bottom of the glass can be calculated when $h = 0$.

$$\therefore d = \frac{0}{5} + 6$$
$$= 6$$

The diameter of the bottom of the glass is 6 cm.

d When $d = 9$, $9 = \frac{h}{5} + 6$
 $\therefore 3 = \frac{h}{5}$
 $\therefore h = 15$

The height of the glass is 15 cm.

2 a When $n = 100$, $C = 108$,

$$\therefore 108 = 100a + b \quad \dots [1]$$

When $n = 120$, $C = 100$,

$$\therefore 100 = 120a + b \quad \dots [2]$$

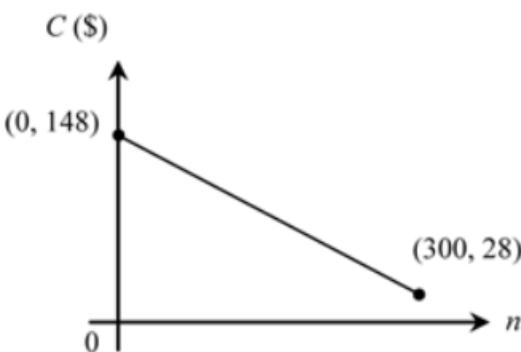
Subtract [2] from [1]

$$8 = -20a$$
$$\therefore a = \frac{-8}{20}$$
$$= -0.4$$

Substitute $a = -0.4$ in [1]

$$108 = 100 \times -0.4 + b$$
$$= -40 + b$$
$$\therefore b = 148$$

b $C = -0.4n + 148$, $0 \leq n \leq 300$



c When $n = 200$, $C = -0.4 \times 200 + 148 = 68$

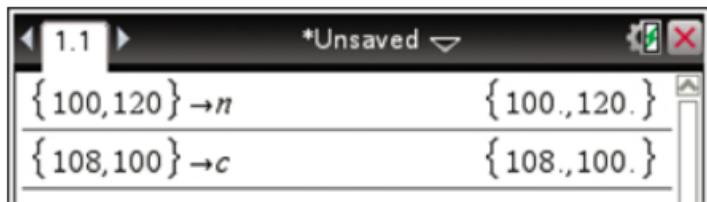
If 200 jackets are made, each jacket will cost \$68 to manufacture.

d When $C = 48.8$, $48.8 = -0.4n + 148$
 $\therefore 0.4n = 99.2$
 $\therefore n = 248$

If the cost of manufacturing each jacket is \$48.80, 248 jackets are produced in the run.

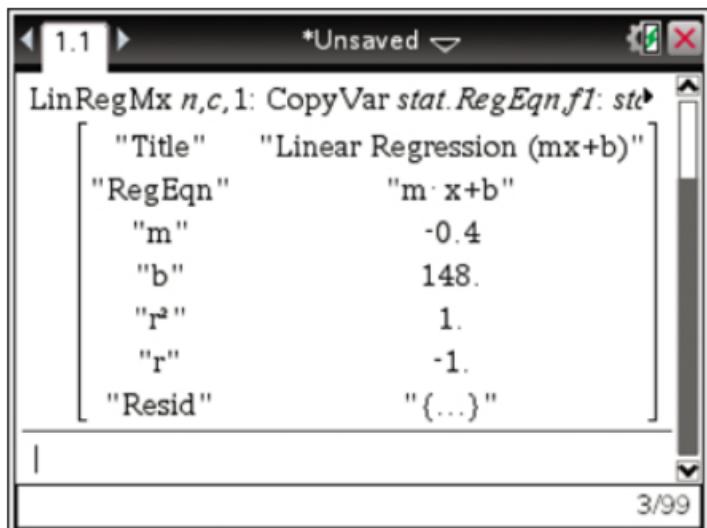
CAS calculator techniques for Question 4

TI: In the Calculator page type $\{100, 120\} \rightarrow n$ then ENTER followed by $\{108, 100\} \rightarrow c$ then ENTER. Press **Menu** → **6: Statistics** → **1: Stat Calculations** → **3:Linear Regression (mx + b)**. Set X List to **n** and Y List to **c**.

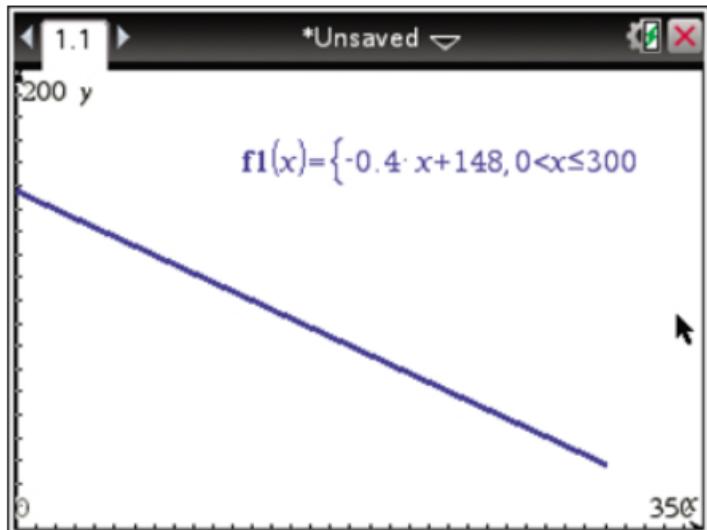


The equation of the line is $C = -0.4n + 148$

In a Graphs page input $-0.4x + 148 | 0 < x \leq 300$ into **f1** then ENTER. In a Calculator page type **f1(200)** to yield a value of 68.



To answer part **d** sketch $f2 = 48.8$. Press **Menu** → **6: Analyze Graph** → **4:Intersection**



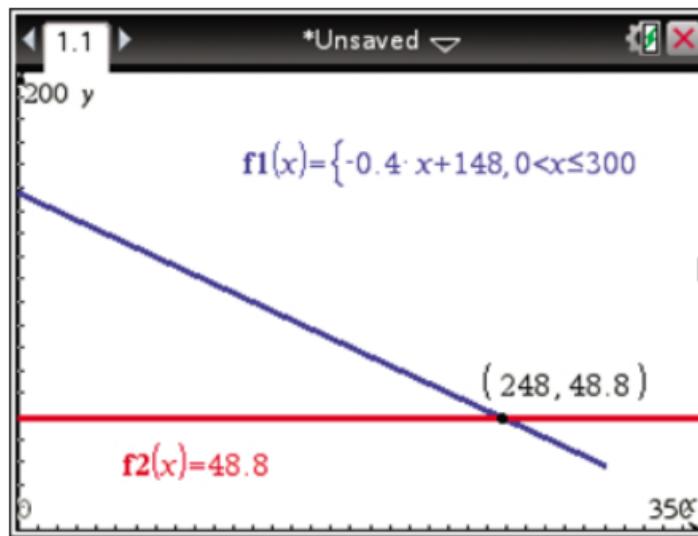
CP: In the Main application type $\{100, 120\} \text{Bn}$ then EXE followed by $\{108, 100\} \text{Bc}$ then EXE. In the (tab of the Keyboard select **LinearReg** and complete the command as **LinearReg n,c** followed by EXE. Tap

Action → **Command** → **DispStat**

The equation of the line is $C = -0.4n + 148$

In a Graph&Table application input

$-0.4x+148 | 0 < x \leq 300$ into **y1** then EXE. Tap \$ then **Analysis** → **G-Solve** → **y – Cal** and input 200 as the x-value to yield a value of 68.



To answer part d sketch $y_2 = 48.8$. Tap

Analysis → G-Solve → Intersect

3 a i When $n = 180$, $A = 180 - \frac{360}{180}$
 $= 178$

ii When $n = 360$, $A = 180 - \frac{360}{360}$
 $= 179$

iii When $n = 720$, $A = 180 - \frac{360}{720}$
 $= 179.5$

iv When $n = 7200$, $A = 180 - \frac{360}{7200}$
 $= 179.95$

b i As n becomes very large, A approaches 180.

ii As n becomes very large, the shape of the polygon approaches that of a circle.

c When $A = 162$, $162 = 180 - \frac{360}{n}$
 $\therefore \frac{360}{n} = 18$
 $\therefore n = \frac{360}{18}$
 $= 20$

d $A = 180 - \frac{360}{n}$
 $\therefore \frac{360}{n} = 180 - A$
 $\therefore n = \frac{360}{180 - A}$

e For an octagon, $n = 8$

$$\therefore A = 180 - \frac{360}{8}$$

$$= 135$$

At the point where the two octagons and the third regular polygon meet, the three angles sum to 360° ,

$$\therefore 135 + 135 + x = 360$$

where x° is the size of the interior angle of the third regular polygon.

$$\therefore 270 + x = 360$$

$$\therefore x = 90$$

Thus the third regular polygon is a square.

4 a Volume of hemisphere, $V_H = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi t^3$

Volume of cylinder, $V_{CY} = \pi r^2 h = \pi t^2 s$

Volume of cone, $V_{CO} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi t^2 w$

b i If $V_H = V_{CY} = V_{CO}$

then $\frac{2}{3}\pi t^3 = \pi t^2 s \quad \dots [1]$

and $\pi t^2 s = \frac{1}{3}\pi t^2 w \quad \dots [2]$

From [1] $\frac{t^3}{t^2} = \frac{3}{2}s$

From [2] $w = \frac{\pi t^2 s}{\frac{1}{3}\pi t^2}$

$$= 3s \quad \therefore w : s : t = 3s : s : \frac{3}{2}s$$

$$= 3 : 1 : \frac{3}{2}$$

$$= 6 : 2 : 3$$

ii If $w + s + t = 11$

then $3s + s + \frac{3}{2}s = 11$

$\therefore \frac{11}{2}s = 11$

$\therefore s = 2$

$$w = 3 \times 2$$

$$= 6$$

$$t = \frac{3}{2} \times 2$$

$$= 3$$

Total volume $= V_H + V_{CY} + V_{CO}$

$$= \frac{2}{3}\pi t^3 + \pi t^2 s + \frac{1}{3}\pi t^2 w$$

$$= \frac{2}{3}\pi \times 3^3 + \pi \times 3^2 \times 2 + \frac{1}{3}\pi \times 3^2 \times 6$$

$$= 18\pi + 18\pi + 18\pi$$

$$= 54\pi$$

The total volume is 54π cubic units.

5 a When $n = 1$, $P = -9000$,

$\therefore -9000 = a + b \quad \dots [1]$

When $n = 5$, $P = 15000$

$\therefore 15000 = 5a + b \quad \dots [2]$

Subtract [1] from [2]

$\therefore 24000 = 4a$

$\therefore 6000 = a$

Substitute $a = 6000$ in [1]

$$\therefore -9000 = 6000 + b$$

$$\therefore -15000 = b$$

b $P = 6000n - 15000$

When $n = 12$, $P = 6000 \times 12 - 15000$
 $= 57000$

The profit is \$57000.

c When $P = 45000$, $45000 = 6000n - 15000$

$$\therefore 60000 = 6000n$$
$$\therefore 10 = n$$

The profit will be \$45000 at the end of 2016, after 10 years of operation.

6 a Perimeter of rectangle $= 2(3x + x)$
 $= 8x$

The perimeter of the rectangle is $8x$ cm.

b Perimeter of square = length of wire – perimeter of rectangle
 $= 28 - 8x$

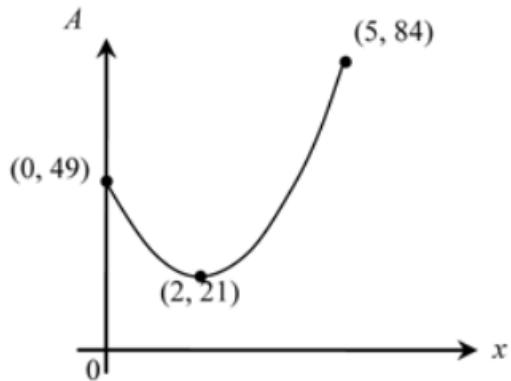
The perimeter of the square is $(28 - 8x)$ cm.

c Side length of square $= \frac{28 - 8x}{4}$
 $= 7 - 2x$

The length of each side of the square is $(7 - 2x)$ cm.

d $A = \text{area of rectangle} + \text{area of square}$
 $= 3x \times x + (7 - 2x)^2$
 $= 3x^2 + 49 - 28x + 4x^2$
 $= 7x^2 - 28x + 49$
 $= 7(x^2 - 4x + 7)$ as required .

e



f $A = 7x^2 - 28x + 49$

Minimum value occurs at $x = \frac{-b}{2a}$, where $a = 7$ and $b = -28$
 $= \frac{28}{14}$
 $= 2$

When $x = 2$, $A = 7(2^2 - 4 \times 2 + 7)$
 $= 21$

A has a minimum value of 21 when $x = 2$.

CAS calculator techniques for Question 9

TI: Sketch the graph of

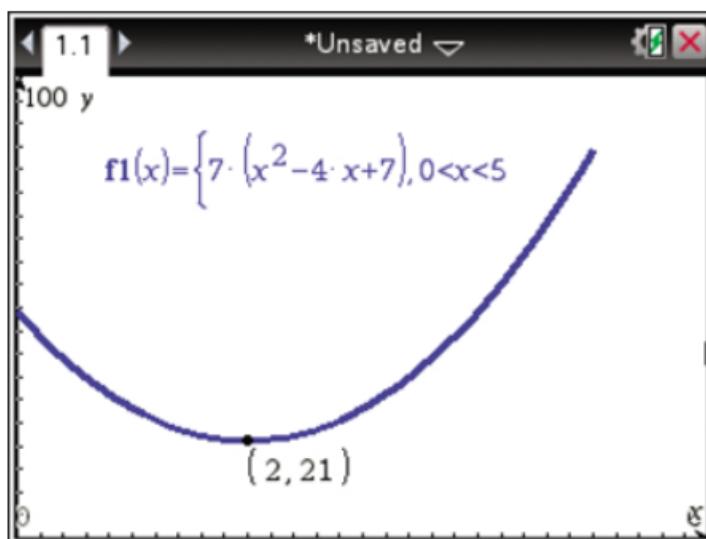
$$f1(x) = 7(x^2 - 4x + 7) \mid 0 < x < 5$$

Press **Menu** → **6: AnalyzeGraph** → **2: Minimum** to yield the minimum value.

CP: Sketch the graph of

$$y1(x) = 7(x^2 - 4x + 7) \mid 0 < x < 5$$

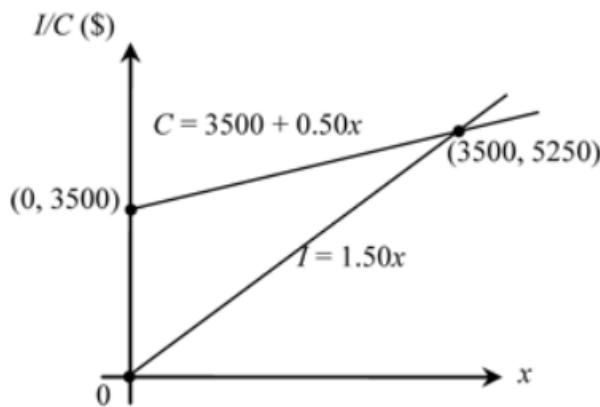
Press **Analysis** → **G-Solve** → **Min** to yield the minimum value.



7 a $C = 3500 + 0.50x$

b $I = 1.50x$

c



d When $I = C$, $1.50x = 3500 + 0.50x$
 $\therefore x = 3500$

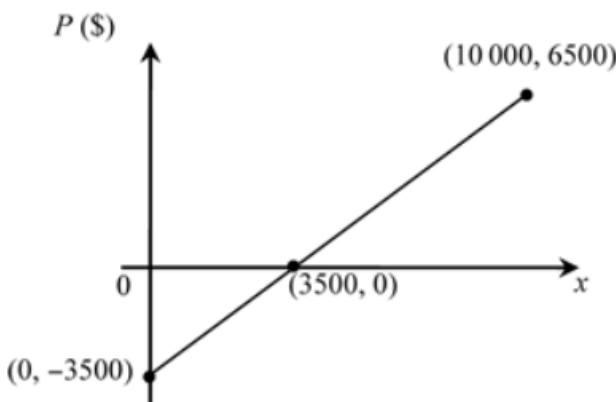
Income equals cost of production when 3500 plates have been sold.

e $I - C = 2000$
 $\therefore 1.50x - (3500 + 0.50x) = 2000$
 $\therefore x - 3500 = 2000$
 $\therefore x = 5500$

A profit of \$2000 is made when 5500 plates are sold.

f $P = I - C$
 $= 1.50x - (3500 + 0.50x)$
 $= x - 3500$

P represents the profit made.



8 a i

$$\begin{aligned}
 & \sqrt{7x-5} - \sqrt{2x} = \sqrt{15-7x} \\
 \Rightarrow & (\sqrt{7x-5} - \sqrt{2x})^2 = (\sqrt{15-7x})^2 \\
 \Rightarrow & (\sqrt{7x-5})^2 - 2(\sqrt{7x-5})(\sqrt{2x}) + (\sqrt{2x})^2 = 15 - 7x \\
 \Rightarrow & 7x - 5 - 2\sqrt{(7x-5)(2x)} + 2x = 15 - 7x \\
 \Rightarrow & 9x - 5 - 2\sqrt{14x^2 - 10x} = 15 - 7x \\
 \Rightarrow & 9x - 5 - 15 + 7x = 2\sqrt{14x^2 - 10x} \\
 \Rightarrow & 16x - 20 = 2\sqrt{14x^2 - 10x} \\
 \Rightarrow & 8x - 10 = \sqrt{14x^2 - 10x}, \text{ as required.}
 \end{aligned}$$

ii

$$\begin{aligned}
 (8x-10)^2 &= (\sqrt{14x^2 - 10x})^2 \\
 \therefore 64x^2 - 160x + 100 &= 14x^2 - 10x \\
 \therefore 64x^2 - 160x + 100 - 14x^2 + 10x &= 0 \\
 \therefore 50x^2 - 150x + 100 &= 0 \\
 \therefore x^2 - 3x + 2 &= 0, \text{ as required.}
 \end{aligned}$$

iii Consider $\sqrt{7x-5} - \sqrt{2x} = \sqrt{15-7x}$

$$\begin{aligned}
 \text{When } x = 1, \text{ LHS} &= \sqrt{7 \times 1 - 5} - \sqrt{2 \times 1} \\
 &= \sqrt{2} - \sqrt{2} = 0 \\
 \text{RHS} &= \sqrt{15 - 7 \times 1} \\
 &= \sqrt{8} \neq 0
 \end{aligned}$$

Hence LHS \neq RHS and $x = 1$ is not a solution.

$$\begin{aligned}
 \text{When } x = 2, \text{ LHS} &= \sqrt{7 \times 2 - 5} - \sqrt{2 \times 2} \\
 &= \sqrt{9} - \sqrt{4} \\
 &= 3 - 2 = 1 \\
 \text{RHS} &= \sqrt{15 - 7 \times 2} \\
 &= \sqrt{1} = 1
 \end{aligned}$$

Hence LHS = RHS and $x = 2$ is a solution.

b i

$$\begin{aligned}
 \sqrt{x+2} - 2\sqrt{x} &= \sqrt{x+1} \\
 (\sqrt{x+2} - 2\sqrt{x})^2 &= (\sqrt{x+1})^2 \\
 x+2 - 4\sqrt{x+2}\sqrt{x} + 4x &= x+1 \\
 5x+2 - 4\sqrt{(x+2)x} &= x+1 \\
 5x+2 - x-1 &= 4\sqrt{x^2+2x} \\
 4x+1 &= 4\sqrt{x^2+2x} \\
 (4x+1)^2 &= (4\sqrt{x^2+2x})^2 \\
 16x^2+8x+1 &= 16(x^2+2x) \\
 16x^2+8x+1 &= 16x^2+32x \\
 1 &= 24x \\
 x &= \frac{1}{24}
 \end{aligned}$$

Consider $\sqrt{x+2} - 2\sqrt{x} = \sqrt{x+1}$

When $x = \frac{1}{24}$, LHS = $\sqrt{\frac{1}{24} + 2} - 2\sqrt{\frac{1}{24}}$
= $\sqrt{\frac{49}{24}} - \frac{2}{2\sqrt{6}}$
= $\frac{7}{2\sqrt{6}} - \frac{2}{2\sqrt{6}} = \frac{5}{2\sqrt{6}}$

and RHS = $\sqrt{\frac{1}{24} + 1}$
= $\sqrt{\frac{25}{24}}$
= $\frac{5}{2\sqrt{6}}$

Hence LHS = RHS and $x = \frac{1}{24}$ is a solution.

ii

$$\begin{aligned} & 2\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x} \\ \Rightarrow & (2\sqrt{x+1} + \sqrt{x-1})^2 = (3\sqrt{x})^2 \\ \Rightarrow & 4(x+1) + 4\sqrt{(x+1)(x-1)} + x-1 = 9x \\ \Rightarrow & 4x+4 + 4\sqrt{(x+1)(x-1)} + x-1 = 9x \\ \Rightarrow & 5x+3 + 4\sqrt{x^2-1} = 9x \\ \Rightarrow & 4\sqrt{x^2-1} = 4x-3 \\ \Rightarrow & (4\sqrt{x^2-1})^2 = (4x-3)^2 \\ \Rightarrow & 16(x^2-1) = 16x^2-24x+9 \\ \Rightarrow & 16x^2-16 = 16x^2-24x+9 \\ \Rightarrow & 24x = 25 \\ \Rightarrow & x = \frac{25}{24} \end{aligned}$$

Consider $2\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x}$

When $x = \frac{25}{24}$, LHS = $2\sqrt{\frac{25}{24} + 1} + \sqrt{\frac{25}{24} - 1}$
= $2\sqrt{\frac{49}{24}} + \sqrt{\frac{1}{24}}$
= $\frac{2 \times 7}{2\sqrt{6}} + \frac{1}{2\sqrt{6}}$
= $\frac{15}{2\sqrt{6}}$

and RHS = $3\sqrt{\frac{25}{24}}$
= $\frac{3 \times 5}{2\sqrt{6}}$
= $\frac{15}{2\sqrt{6}}$

Hence LHS = RHS and $x = \frac{25}{24}$ is a solution.

CAS calculator techniques for Question 12

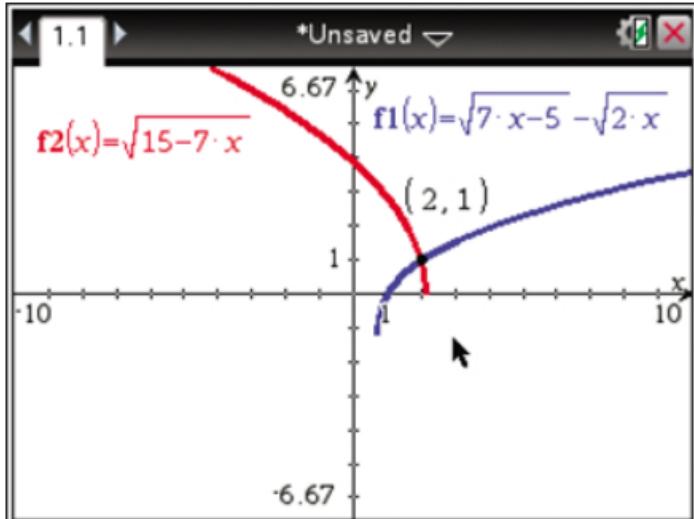
Tl: Sketch the graphs of $f_1 = \sqrt{7x-5} - \sqrt{2x}$

and $f_2 = \sqrt{15-7x}$

Press Menu → 6: Analyze

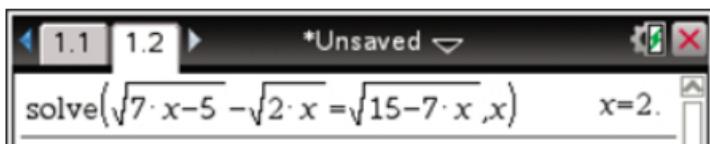
Graph → 4: Intersection

CP: Sketch the graphs of $y_1 = \sqrt{7x - 5} - \sqrt{2x}$ and $y_2 = \sqrt{15 - 7x}$ Tap Analysis → G-Solve → Intersect



Alternatively, type

solve ($\sqrt{7x - 5} - \sqrt{2x} = \sqrt{15 - 7x}, x$) in a Calculator/Main page.



9 a $n + 25$ is a perfect square implies

$$n + 25 = b^2$$

$$\therefore n = b^2 - 25$$

$$= (b - 5)(b + 5)$$

$$\text{Let } a = b - 5$$

$$\text{then } b + 5 = a + 10$$

$$\therefore n = a(a + 10)$$

b Note: $0 < a(a + 10) < 50$

$$\therefore a(a + 10) - 50 < 0 \quad \dots [1]$$

$$\text{and} \quad a(a + 10) > 0 \quad \dots [2]$$

$$\text{From [1]} \quad a^2 + 10a + 25 - 75 < 0$$

$$\therefore (a + 5)^2 - (5\sqrt{3})^2 < 0$$

$$\therefore (a + 5 - 5\sqrt{3})(a + 5 + 5\sqrt{3}) < 0$$

$$\therefore a < -5 + 5\sqrt{3} \text{ and } a > -5 - 5\sqrt{3}$$

$$\text{From [2], } a < -10 \text{ or } a > 0$$

$$\therefore a = 3 \text{ or } 2 \text{ or } 1 \text{ or } -13 \text{ or } -12 \text{ or } -11$$

Consider $10p + q = a^2 + 10a$.

$$a = 1, p = 1, q = 1$$

$$a = 2, p = 2, q = 4$$

$$a = 3, p = 3, q = 9$$

$$a = -11, p = 1, q = 1$$

$$a = -12, p = 2, q = 4$$

$$a = -13, p = 3, q = 9$$

Hence $q = p^2$.

c From the above, $n = 11$ or 24 or 39 .

10a

$$\therefore P = mgh \text{ for a constant } g \in R \setminus \{0\}$$

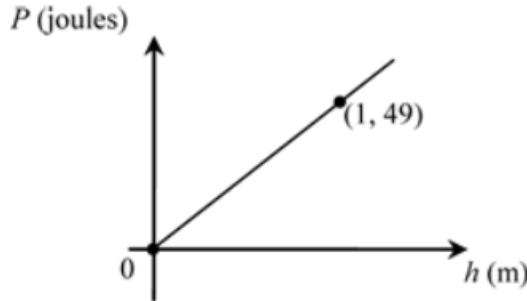
When $m = 5$, $P = 5gh$

$$\therefore g = \frac{P}{5h}$$

i When $P = 980$, $h = 20$,

$$g = \frac{980}{5 \times 20} \\ = 9.8$$

ii



iii When $h = 23.2$,

$$m = 7$$

b i Let $P_1 = 9.8mh$,

$$\therefore P_2 = 9.8m \times (2h) \\ = 19.6mh \\ = 2P_1$$

$$\begin{aligned}\text{Percentage change in potential energy} &= \frac{P_2 - P_1}{P_1} \times 100 \\ &= \frac{2P_1 - P_1}{P_1} \times 100 \\ &= 100\end{aligned}$$

The potential energy has increased by 100%.

ii Let $P_1 = 9.8mh$

$$\therefore P_2 = 9.8 \times 2m \times \frac{1}{4}h \\ = 4.9mh \\ = \frac{1}{2}P_1$$

$$\begin{aligned}\text{Percentage change in potential energy} &= \frac{P_2 - P_1}{P_1} \times 100 \\ &= \frac{\frac{1}{2}P_1 - P_1}{P_1} \times 100 \\ &= -50\end{aligned}$$

The potential energy has decreased by 50%.

c i When $h = 10$,

$$V = \sqrt{19.6 \times 10} \\ = 14$$

ii When $h = 90$,

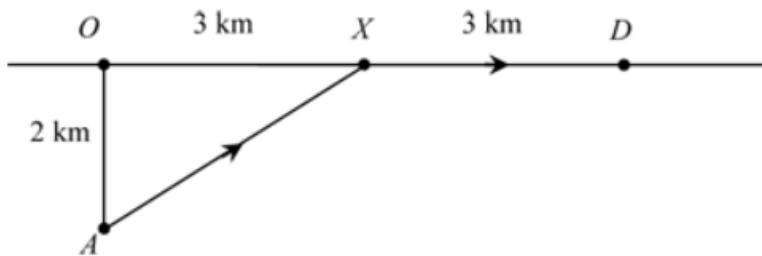
$$V = \sqrt{19.6 \times 90} \\ = 42$$

d Let $V_1 = \sqrt{19.6h_1}$

$$\begin{aligned}\therefore V_2 &= 2V_1 \\ &= 2\sqrt{19.6h_1} \\ &= \sqrt{19.6 \times 4h_1} \\ &= \sqrt{19.6h_2} \text{ where } h_2 = 4h_1\end{aligned}$$

The height must be increased by a factor of 4.

11a



From the diagram,

$$\begin{aligned}AX &= \sqrt{2^2 + 3^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13}\end{aligned}$$

Distance travelled = speed × time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\therefore \text{Time taken for } AX = \frac{\sqrt{13}}{3}$$

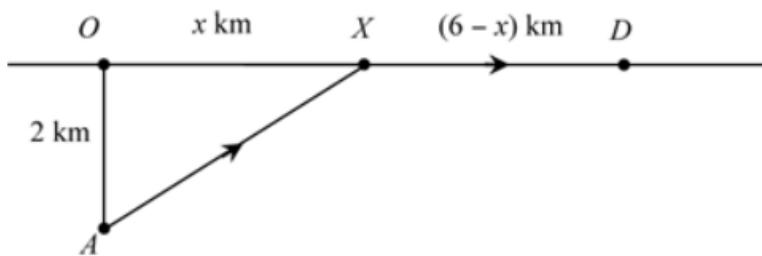
$$\text{Time taken for } XD = \frac{3}{8}$$

$$\begin{aligned}\text{Total time taken} &= \frac{\sqrt{13}}{3} + \frac{3}{8} \\ &= 1.57685\dots\end{aligned}$$

$$\begin{aligned}1.57685\dots \text{ hours} &= 1 \text{ hour and } 0.57685\dots \times 60 \text{ minutes} \\ &= 1 \text{ hour } 34.61102\dots \text{ minutes}\end{aligned}$$

The time taken was 1 hour 35 minutes, correct to the nearest minute.

b



$$\begin{aligned}\text{From the diagram, } AX &= \sqrt{2^2 + x^2} \\ &= \sqrt{x^2 + 4}\end{aligned}$$

Off-road he walks at 3 km/h

$$\therefore \text{Time taken for } AX = \frac{\sqrt{x^2 + 4}}{3}$$

On-road he walks at 8 km/h for a distance of $(6 - x)$ km

$$\therefore \text{Time taken for } XD = \frac{6-x}{8}$$

$$\text{Total time taken} = \frac{\sqrt{x^2 + 4}}{3} + \frac{6-x}{8} = \frac{3}{2}$$

$$\therefore 8\sqrt{x^2 + 4} + 3(6 - x) = 36$$

$$\therefore 8\sqrt{x^2 + 4} + 18 - 3x = 36$$

$$\therefore 8\sqrt{x^2 + 4} = 3x + 18$$

$$\therefore (8\sqrt{x^2 + 4})^2 = (3x + 18)^2$$

$$\therefore 64(x^2 + 4) = 9x^2 + 108x + 324$$

$$\therefore 64x^2 + 256 = 9x^2 + 108x + 324$$

$$\therefore 55x^2 - 108x - 68 = 0$$

$$x = \frac{+108 \pm \sqrt{(-108)^2 - 4 \times 55 \times (-68)}}{2 \times 55}$$

$$= -0.50153\dots, 2.46516\dots$$

but $x > 0$, $\therefore x = 2.46516\dots$

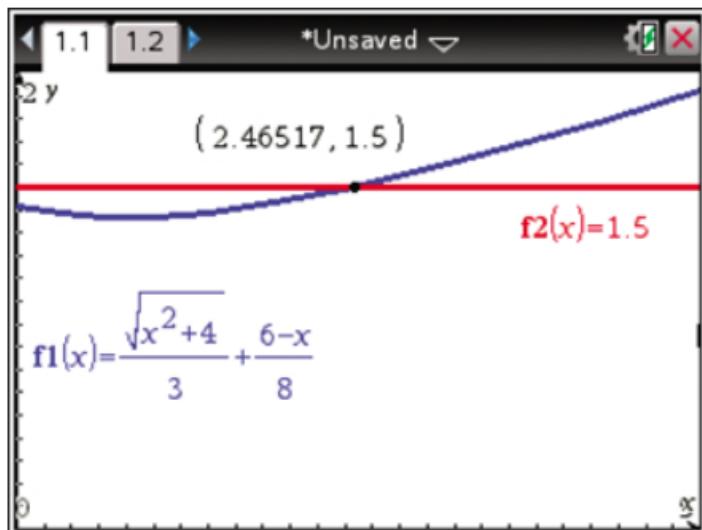
If the total time taken was $1\frac{1}{2}$ hours, OX is 2.5 km correct to one decimal place.

CAS calculator techniques for Question 19

Sketch the graphs of $f1(x) = \frac{\sqrt{x^2 + 4}}{3} + \frac{6-x}{8}$ and $f2(x) = 1.5$

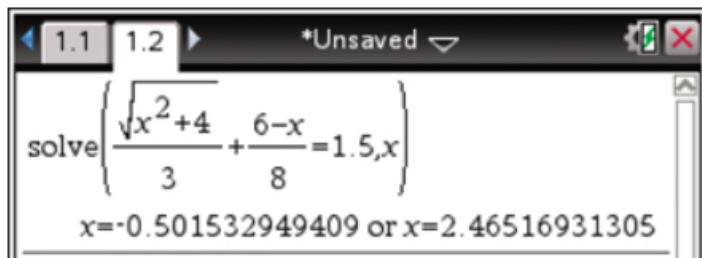
T1: Press Menu → 6: Analyze Graph → 4: Intersection

CP: Tap Analysis → G-Solve → Intersect



Alternatively, type

$\text{solve}\left(\frac{\sqrt{x^2 + 4}}{3} + \frac{6-x}{8} = 1.5, x\right)$ and interpret answers recalling $x > 0$.



12a

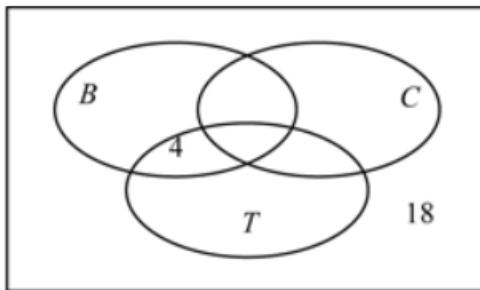
$$|B' \cap C' \cap T| = |C \cap T|$$

$$|B \cap C' \cap T'| = 3|B' \cap C \cap T'|$$

$$|B \cap C' \cap T| = 4$$

b

$$n(\xi) = 76$$



$$|C \cap T| + |B' \cap C' \cap T| + |B \cap C' \cap T| = |T|$$

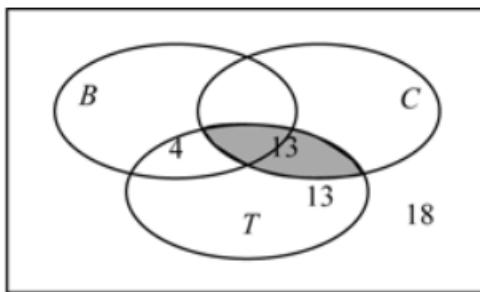
$$\therefore 2|C \cap T| + 4 = 30 \text{ as } |C \cap T| = |B' \cap C' \cap T|$$

$$\therefore |C \cap T| = \frac{30 - 4}{2}$$

$$= 13$$

$$\therefore |B' \cap C' \cap T| = 13$$

$$n(\xi) = 76$$



$$\text{Let } |B' \cap C \cap T| = y$$

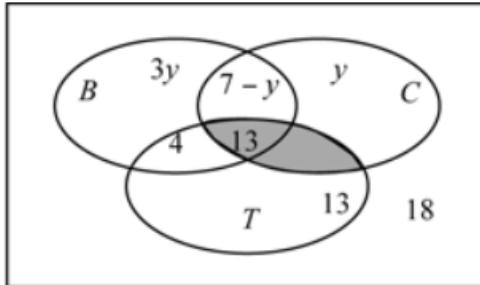
$$|B \cap C' \cap T'| = 3y$$

$$|C| = |B' \cap C \cap T'| + |C \cap T| + |B \cap C \cap T'|$$

$$\therefore 20 = y + 13 + |B \cap C \cap T'|$$

$$\therefore |B \cap C \cap T'| = 7 - y$$

$$n(\xi) = 76$$



$$\text{Now } 3y + (7 - y) + 4 + 13 + 13 + y + 18 = 76$$

$$\therefore 3y + 55 = 76$$

$$\therefore 3y = 21$$

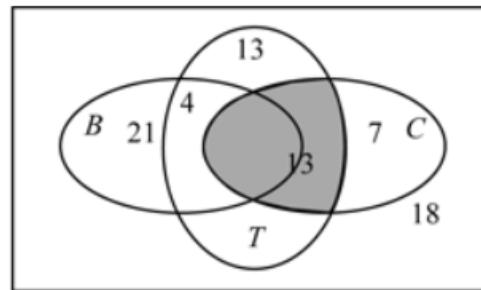
$$\therefore y = 7$$

$$|B' \cap C \cap T'| = 7$$

$$|B \cap C' \cap T'| = 21$$

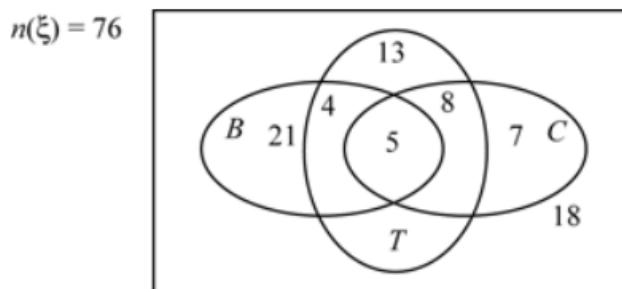
$$|B \cap C \cap T'| = 0$$

$$n(\xi) = 76$$



$$\begin{aligned}|B \cap C \cap T| &= |B| - |B \cap C' \cap T'| - |B \cap C' \cap T| \\&= 30 - 21 - 4 \\&= 5\end{aligned}$$

$$\therefore |B' \cap C \cap T| = 13 - 5 = 8$$



c i $|B \cap C \cap T| = 5$

ii $|B' \cap C \cap T'| = 8$